mathematical models

The importance of mathematical models to scientific discovery

A case study on the feeding mechanism of the Goliath Grouper *Epinephelus itajara*

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Introduction

The use of collaborative problem solving within mathematics education is imperative in this day and age of integrative science. The formation of interdisciplinary teams of mathematicians and scientists to investigate crucial problems is on the rise, as greater insight can be gained from an interdisciplinary perspective. Mathematical modelling, in particular, is increasingly recognised as a fundamental tool in understanding scientific phenomena, with models utilising mathematical disciplines ranging from statistics to differential equations (Giordano, Fox & Horton, 2014). Geometry is an effective tool in biomechanical modelling, and one that we have used to develop a series of lessons regarding the functional importance of mathematics in nature (see http://utweb.ut.edu/rwaggett/science-math-master.html). In this manuscript, we present a lesson in which geometry is used to model the suction feeding mechanism of the Goliath Grouper, and the consequences of geometric variability for organismal performance are explored.

Fish feeding mechanisms

The great diversity of skull morphology amongst vertebrates is associated with the varied food resources that they consume. During the course of evolutionary history there has been a trend towards the reduction of bones and mobility in the vertebrate skull, resulting in more stable structures capable of generating and resisting greater force during feeding, particularly in the case of the terrestrial vertebrates. Conversely, fish skulls are highly kinetic and composed of numerous moving parts; the average fish skull contains about 50 bones with at least 7 moving parts, whereas the human skull has only 22 bones and a single moving part (i.e. lower jaw) (Liem, Bemis, Walker, & Grande, 2001). Given the tremendous diversity of fish and their complex cranial structures, there is also a considerable diversity of fish feeding mechanisms, including filter-, biting-, ram-, and suction-feeding.

Filter feeding entails the forward motion of an open-mouthed fish, such as a whale shark *Rhincodon typus*, through the water column from which planktonic prey are captured on highly modified gill structures and subsequently transported to the digestive system (Motta et al., 2010). Biting involves the use of the jaws to capture and process prey, and is usually associated with the consumption of hard, benthic prey that has limited escape ability. Fish that use biting, such as the horn shark *Heterodontus francisci,* usually have forceful, high- leverage jaws with pavement-like teeth for crushing and grinding food (Motta & Huber, 2012). Ram feeding involves the predator overtaking its prey by swimming faster than the prey can escape, and is usually associated with fish that have fast jaws and sharp teeth, such as the great barracuda *Sphyraena barracuda* (Habegger, Motta, Huber, & Deban, 2011). Finally, suction feeding is the most common mechanism and believed to be the original fish feeding mechanism in evolutionary history. It entails the rapid expansion of the mouth and gill region, which creates a negative pressure that causes water to flow into the mouth, along with any prey items that are located within the moving water. Suction pressure and

water flow velocity are dependent on the magnitude and rate of volume change, as well as the cross-sectional area of the mouth (Bishop, Wainwright & Holzman, 2008). Fish that generate a lot of expansion in a short period of time, such as the largemouth bass *Micropterus salmoides*, can capture their prey in less than a tenth of a second (Svanback, Wainwright, & Ferry-Graham, 2002).

Geometric modelling of suction feeding

Sometimes the best science happens when researchers find very simple models to explain very complicated events. One such example is suction feeding, in which the head can be modelled as a series of expanding cones (Van Wassenbergh, Herrel, Adriaens, & Aerts, 2007, and Bishop et al., 2008). The purpose of this exercise is to use a simple model (one full cone + one truncated cone) of the feeding mechanism of the Goliath Grouper to determine:

1. the water flow velocity that the Goliath Grouper generates during suction feeding, and
2. how suction feeding by the Goliath Grouper compares to that of other fish.

We suggest teaching this lesson during a unit on three-dimensional geometric shapes, and using three class periods; two for establishing the context and mathematical principles of the investigation and one for performing the model calculations.

Class 1

Introduce the concepts of a cone and a truncated cone with three-dimensional models that students can examine. Provide simple examples of cones and truncated cones with measurements, have students label two-dimensional drawings with these measurements, and compute the volume of each cone and truncated cone by extending the truncated cone to a full cone and then subtracting the volume of the extension from the total volume.

Class 2

First, engage students by having them investigate the Goliath Grouper *Epinephelus itajara*, and the two fish with which it will be compared: the snook Centropomis undecimalis, and the longjaw butterfly fish *Forcipiger longirostris*. Pictures and information for this lesson are available on our website (mentioned previously), including PowerPoint slides and an aquarium video tour.

Figure 1. Goliath Grouper *Epinephelus itajara*, Snook *Centropomis undecimalis*, and Longjaw Butterfly Fish *Forcipiger longirostris*.

Discuss the obvious differences in geometry of the feeding mechanisms of three fish using the provided materials. Spend time explaining that the ability of an animal to perform a task such as suction feeding, is contingent upon the behavior of that animal, which in this case is accounted for using the time each species takes to expand its feeding mechanism. This is a great way to establish that the volume change of the feeding mechanism does not need

to be tremendous if the fish moves its feeding mechanism very quickly: effective suction feeding can be done using large, slow movements of the feeding mechanism or small, fast movements. Students can speculate as to which fish will generate the greatest velocity during suction feeding.

Class 3

Work through the Goliath Grouper worksheet together, and then have students work through the same questions using the snook and longjaw butterfly fish data independently.

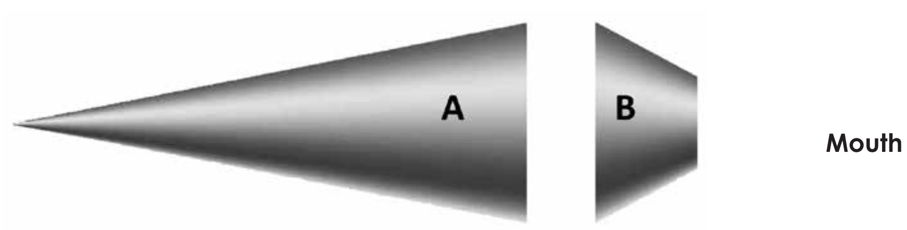
Goliath Grouper worksheet

Goals:

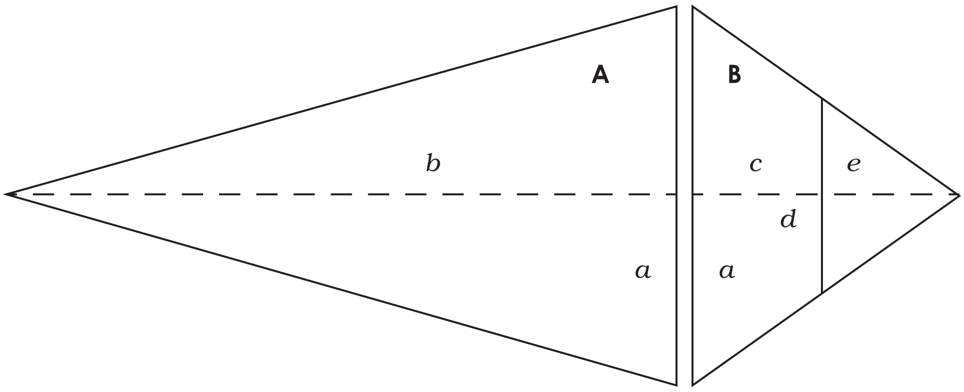
1. Find the volume of the feeding mechanism of the Goliath Grouper at rest.
2. Find the volume of the feeding mechanism of the Goliath Grouper at maximum expansion.
3. Find the area of the mouth of the Goliath Grouper at maximum expansion.
4. Taking the formula as given below, determine the velocity of water flow into the mouth of the Goliath Grouper :

$$\text{Velocity} = \frac{\text{Change in Volume}}{\text{Change in Time}} \times \frac{1}{\text{Area of Mouth}}$$

The feeding mechanism of the Goliath Grouper can be represented by two cones. A full cone (A) extends from beneath the eye to the back of the throat, while a truncated cone (B), extends from beneath the eye to the front opening of the mouth.



Cones A and B can be represented two-dimensionally and used to determine the volume of the Goliath Grouper 's feeding mechanism, which is the first step in calculating the velocity of water flow into its mouth during suction feeding. To do so, volume calculations must be done with the feeding mechanism at rest and at maximum expansion (Note: an extension is shown to help you find the area of the truncated cone).



a = radii of cones A and B
b = height of cone A
c = height of truncated portion of cone B
d = radius of extended portion of cone B
e = height of extended portion of cone B

Suction feeding exercise

A. Procedure for mathematical calculations

Compute answers for the feeding mechanism at rest.
a = 34.9 mm
b = 153.9 mm
c = 54.3 mm
d = 6.4 mm

Step 1: Compute the volume of cone A.

$$V = \frac{1}{3}\pi r^2 h$$

Step 2: Find e (the height of the extended portion of cone B) using similar triangles.
Step 3: Find the total height of cone B (extended portion + truncated portion).
Step 4: Find the total volume of cone B (extended portion + truncated portion).
Step 5: Find the volume of the extended portion of cone B, which is also a cone.
Step 6: Find the volume of the truncated portion of cone B.
Step 7: Find the volume of the feeding mechanism at rest (time t_0).

B. Exercises to complete

i. Compute answers for the feeding mechanism when fully expanded (time t_1). Repeat steps one through seven using the measurements for the feeding mechanism when fully expanded.
a = 39.5 mm
b = 161.3 mm
c = 56.4 mm
d = 32.6 mm

ii. Calculate the area of the mouth of the Goliath Grouper when fully expanded.

$$\text{Area} = \pi r^2$$

iii. Find the change in volume of the feeding mechanism during the feeding event.

iv. Compute the velocity of water flow into the mouth given that it took 0.132 seconds (sec) for the Goliath Grouper to expand its feeding mechanism.

$$\text{Velocity} = \frac{\text{Change in Volume}}{\text{Change in Time}} \times \frac{1}{\text{Area of Mouth}}$$

Solution: 486.3 mm/sec

Snook Data

Measurements for the feeding mechanism at rest (time t_0):
a = 2.1 mm
b = 27.6 mm
c = 12.3 mm
d = 1.8 mm

Measurements for the feeding mechanism when fully expanded (time t_1):
a = 7.0 mm
b = 28.9 mm
c = 12.3 mm
d = 5.9 mm
Duration of feeding event: 0.036 sec
Solution: 716.2 mm/sec

Longnose Butterfly Fish Data

Measurements for the feeding mechanism at rest (time t_0):
a = 5.0 mm
b = 14.9 mm
c = 31.2 mm
d = 1.1 mm

Measurements for the feeding mechanism when fully expanded (time t_1):
a = 5.0 mm
b = 14.9 mm
c = 31.6 mm
d = 1.1 mm
Duration of feeding event: 0.022 sec
Solution: 158.8 mm/sec

Conclusion

Having performed these calculations for all three species, students will find that while the snook generates the greatest velocity, there are very different strategies, all of which can result in high performance suction feeding. The Goliath Grouper has a tremendous increase in volume, but does so relatively slowly (0.132 sec), whereas the longnose butterfly fish has a tiny increase in volume, but does so very rapidly (0.022 sec).

This work was supported by a grant from the Florida Department of Education.

References

Bishop, K.L., Wainwright, P.C. & Holzman, R. (2008). Anterior to Posterior Wave of Buccal Expansion in Suction Feeding Fish is Critical for Optimizing Fluid Flow Velocity Profile. *Interface: Journal of the Royal Society, 5*, 1309–1316.

Giordano, F.R., Fox, W.P. & Horton, S.B. (2014). *A First Course in Mathematical Modeling.* Boston, MA: Brooks/Cole.

Habegger, M.L, Motta, P.J., Huber, D.R. & Deban, S. (2011). Feeding Biomechanics in the

Great Barracuda (Sphyraena barracuda) During Ontogeny. *Journal of Zoology, 283* (1), 63–72.

Liem, K.F., Bemis, W.E., Walker, W.F., Jr. & Grande, L. (2001). *Functional Anatomy of The Vertebrates: An Evolutionary Perspective*. Belmont, CA: Brooks/Cole-Thomson Learning.

Motta, P.J. & Huber, D.R. (2012). Prey Capture Behavior and Feeding Mechanics of Elasmobranchs. In J. Carrier, J. Musick, & M. Heithaus, *Biology of Sharks and Their Relatives.* Boca Raton, FL: CRC Press LLC.

Motta, P.J., Maslanka, M., Hueter, R.E., Davis, R.L., de la Parra, R., Mulvany, S.L., & Zeigler, L.D. (2010). Feeding Anatomy, Filter-Feeding Rate, and Diet of Whale Sharks Rhincodon typus During Surface Ram Filter Feeding off the Yucatan Peninsula, Mexico. *Zoology, 113*, 199–212.

Svanback, R., Wainwright, P.C. & Ferry-Graham, L.A. (2002). Linking Cranial Kinematics,

Buccal Pressure, and Suction Feeding Performance in Largemouth Bass. *Physiological and Biochemical Zoology, 75*(6), 532-543.

Van Wassenbergh, S., Herrel, A., Adriaens, D. & Aerts, P. (2007). No Trade-Off Between Biting and Suction Feeding Performance in Clariid Catfishes. *Journal of Experimental Biology, 210,* 27-36.

Waggett, R. (n.d.). *Science Math Master.* The University of Tampa, Florida. Retrieved 5 August 2015 from http://utweb.ut.edu/rwaggett/science-math-master.html